

# On the Classification Scheme for Phenomenological Universalities in Growth Problems in Physics and Other Sciences

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## Abstract

Comment on "Classification Scheme for Phenomenological Universalities in Growth Problems in Physics and Other Sciences" by P. Castorina, P. P. Delsanto and C. Guiot, Phys. Rev. Lett. **96**, 188701 (2006) is presented. It has been proved that the West-like function of growth derived by the authors is incorrect and the approach does not take into account the growth of the biological systems undergoing atrophy or demographic and economic systems undergoing involution or regression. A simple extension of the model, which permits derivation of the so far unknown involuted Gompertz function of growth is proposed.

The concept of *phenomenological universalities* introduced recently by Castorina, Delsanto and Guiot (CDG) [1] is a useful tool for investigation of the nonlinear processes in complex systems whose dynamics is governed by the system of equations

$$\frac{dy(\tau)}{d\tau} = x(\tau)y(\tau) \qquad \frac{dx(\tau)}{d\tau} = -\Phi(x). \qquad (1)$$

Here,  $\tau = x(0)t$  denotes dimensionless temporal variable, whereas  $\Phi(x)$  is a generating function, which extended into a series of x-variable generates different functions of growth for a variety of patterns emerging in complex systems in physics, biology and beyond. For example for  $\Phi = x$  one gets the Gompertz function, whereas for  $\Phi = x + bx^2$  the allometric West-like function is derived [1]

$$y(\tau)_G = \exp[1 - \exp(-\tau)], \quad y(\tau)_W = [1 + b - b \exp(-\tau)]^{1/(1-b)}. \quad (2)$$

Unfortunately, the West-like function obtained by CDG [1] is incorrect as it is not a solution of the differential equation (9) in [1]. Additionally in the limit  $\lim_{b \rightarrow 0} y(\tau)_W = 1$  it does not produce the Gompertz function as indicated by the authors [1]. To explain those inconsistencies Eqs. (1) were solved for  $\Phi = x + bx^2$  employing Maple vs. 7.0 processor for symbolic calculations. The calculations provided the correct solution  $y(\tau)_W = [1 + b - b \exp(-\tau)]^{1/b}$  with power  $1/b$  and not  $1/(1-b)$  as derived by CDG [1]. Employing the Maple one may also prove that

$$\lim_{b \rightarrow 0} [1 + b - b \exp(-\tau)]^{1/b} = \exp[1 - \exp(-\tau)] \quad (3)$$

as it should be.

Although CDG claimed [1] that they *.....have developed a simple scheme that allows the classification of all the growth problems.....* described by Eqs. (1) this approach does not take into account the growth of the biological systems undergoing atrophy or demographic and economic systems undergoing involution or regression. In biological systems such a situation appears in avian primary lymphoid organs: thymus and bursa of Fabricius as well as in thymus of mammalians. To extend the CDG approach and derive an involuted function of growth, we employ a linear expansion of the generating function  $\Phi(x) = c_1x + c_0$ , which includes a constant term  $c_0$  omitted in the CDG scheme and  $c_1$  coefficient, which in the previous approach was constrained  $c_1 = 1$ . Additionally, we assume that  $x(\tau = 0) = (1 - c_0)/c_1$ , which for  $c_0 = 0$  and  $c_1 = 1$  gives the CDG condition  $x(0) = 1$ . Employing the relationships (1) one gets the involuted Gompertz function ( $c_0, c_1 > 0$ )

$$y(\tau) = \exp \left\{ \frac{1}{c_1^2} [1 - \exp(-c_1\tau)] \right\} \exp \left( -\frac{c_0}{c_1} \tau \right), \quad (4)$$

which can be specified in the form applicable to direct fitting of the experi-

mental data

$$y(t) = y_0 \exp \left\{ \frac{b}{a} [1 - \exp(-at)] \right\} \exp(-bct). \quad (5)$$

Here, we use the correspondences  $c_1^2 = a/b$ ,  $c_1\tau = at$  and  $c = c_0$ . To prove that function (5) correctly describes the evolution and involution of organs undergoing atrophy, we employed it to fit 30 mean absolute weights of the thymus of male Wistar rats evaluated in the period of 1-780 days. In the calculations we used thymuses sampled from strain rats of the mean age of 24 months, originating from randomly mated culture (Department of Toxicology, University of Medical Science in Poznań, Poland). The mean values of the absolute thymus weight were fitted to the 4-parameteric function (5) using the weighted nonlinear least-square routine with statistical weights taken as inverse squares of uncertainties  $u(i)$  of the thymus weights. As the criteria of goodness of the fit, the standard deviation  $\sigma$  and normalized standard deviation  $\hat{\sigma}$  were employed. The calculations provided the values of the parameters  $y_0 = 0.0105(16)$  [g],  $a = 0.0472(44)$  [1/day],  $b = 0.1914(194)$  [1/day] and  $c = 0.0140(38)$  [1/day] reproducing the experimental data with  $\sigma = 0.011$  [g] and  $\hat{\sigma} = 1.64$ . All parameters fitted are statistically well evaluated and the correlation coefficients between them cover a satisfactory range (0.6455 – 0.8971).

The results obtained indicate that the  $U_1$  class introduced by CDG [1] should be divided into two subclasses  $U_{11}$  - representing the sigmoidal Gompertzian growth, and  $U_{12}$  - including the involuted Gompertzian growth. The  $U_{12}$  solution (5) is a very important contribution to the classes obtained previously as it is useful to fit the data for the systems undergoing atrophy. The sigmoidal (S-shaped) functions (Gompertz, West, logistic, Richards etc.) cannot be applied to this aim as they correctly describe the ideal situation in which exponential growth is exponentially retarded and saturated as time continues.

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## References

- [1] P. Castorina, P. P. Delsanto and C. Guiot, Phys. Rev. Lett. **96**, 188701 (2006).